

Universal hydrodynamics and charged hadron multiplicity at the LHC

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Time evolution of a "little bang" created in heavy ion collisions can be divided into two phases, the pre-equilibrium and hydrodynamic. At what moment the evolution becomes hydrodynamic and is there any universality in the hydrodynamic flow? To answer these questions we briefly discuss various versions of hydrodynamics and their applicability conditions. In particular, we elaborate on the idea of "universal" (all-order resummed) hydrodynamics and propose a simple new model for it. The model is motivated by results obtained recently via the AdS/CFT correspondence. Finally, charged hadron multiplicities in heavy ion collisions at the RHIC and LHC are discussed. At the freezeout, the multiplicities can be related to total entropy produced in the collision. Assuming the universal hydrodynamics to hold, we calculate the entropy production in the hydro stage of the collision. We end up speculating about a connection between the multiplicity growth and the temperature dependence of the QGP viscosity.

I. INTRODUCTION

This paper contains some further developments of the ideas put forward in our paper [1]. There we argued that entropy production in the strongly coupled quark gluon plasma (sQGP) should be computed using an all-order resummed hydrodynamics and that the resummation makes it possible to provide reliable estimates even starting from very short thermalization times. The main goal of this note is to connect this proposal to some recent theory developments based on the AdS/CFT setting [2], which support our ideas, as well as to address the phenomenological question of charged particle multiplicity production in heavy ion collisions at the LHC, to be detailed below in section II. Let us stress here that the entropy production is only one of several applications, for which an all-order resummation might be important. There are additional interesting phenomena, in which matter gradients are large and applicability limits of standard hydrodynamics is in question. Let us give here two examples of those.

As recent studies have shown, fluctuations of initial state density in heavy ion collisions are the origin of sound waves. By freezeout, these waves reach large distances, comparable to the fireball radius itself, and are observed as fluctuations of angular harmonics in the particle distributions. It is remarkable that amplitudes of up to 9-th harmonics have been measured, displaying good agreement with hydrodynamics [3–5]. Yet, the questions how to treat these fluctuations in non-equilibrium and from what initial times can they be evolved hydrodynamically remains unanswered.

"Mach cones" induced in the matter by quenching jets [6, 7] present another application of the sound waves in heavy ion physics. Unlike sounds from the previous example, the jet-induced waves were studied in detail within the AdS/CFT context [8]. The results were shown to have a good agreement with hydrodynamics at later stages but when exactly hydro becomes applicable and why still could have been studied more, given the exact AdS/CFT solution. The issue becomes even more important with the first LHC data on jets, revealing events with huge amounts of energy, $\sim 100 \text{ GeV}$, deposited by a jet. This calls for studies of the full nonlinear settings, beyond the linearized sound wave approximation.

In section III, we discuss initial conditions for hydrodynamics from the perspective of the AdS/CFT results. We also propose in this section a new, all-order resummed, hydrodynamics model for Bjorken explosion. In Section IV, we use this model in order to compute the entropy production in the hydro phase. Phenomenological relevance to the data on charged particle multiplicities is also discussed. We summarize and provide additional discussions in section V.

II. MULTIPLICITIES IN pp AND AA COLLISIONS

One of the first discoveries made by the LHC is a rapid rise with energy of multiplicities of charged hadrons produced both in pp and heavy ion collisions. The discovery is especially dramatic in heavy ions collisions, where most of the existing models have failed to predict the data.

The first ALICE data on charged particle multiplicity in lead lead collisions are [9]:

$$\frac{dN}{d\eta}|_{PbPb(2.76TeV)} = 1584 \pm 76, \quad (II.1)$$

combined with the earlier data from the RHIC, these ones imply the multiplicity in AA collisions growing with the (center of mass) energy per nucleon as

$$\frac{dN}{d\eta}|_{PbPb(E_{NN})} \sim E_{NN}^{0.30}. \quad (II.2)$$

The corresponding power in the pp collisions is 0.22, and thus the ratio of the two also grows with the energy

$$\frac{dN}{d\eta}|_{PbPb} / \frac{dN}{d\eta}|_{pp} \sim E_{NN}^{0.08}. \quad (II.3)$$

From the RHIC energy ($E = 0.2TeV$) to the LHC, the double ratio is

$$\frac{\frac{dN}{d\eta}|_{PbPb,LHC} / \frac{dN}{d\eta}|_{pp,LHC}}{\frac{dN}{d\eta}|_{AuAu,RHIC} / \frac{dN}{d\eta}|_{pp,RHIC}} = 1.23. \quad (II.4)$$

This noticeable change with the energy calls for a theoretical explanation. (An increase in the atomic number, 197 for Au and 208 for Pb, explains only 0.055 of it.)

Particle production in heavy ion collisions proceed via two basic phases: (i) a pre-thermalization phase and (ii) a hydrodynamical stage. Theoretical frameworks used for their descriptions are very different.

The first one is based on pQCD cascade of gluons, described by high energy evolution equations including gluon saturation effects, or color glass condensate (CGC). CGC relies on emergence of a semi-hard scale, the saturation momentum

$$Q_s \sim A^{1/3} x^{-\lambda}, \quad \lambda = 0.25 - 0.30 \quad (II.5)$$

related to the density of gluons with longitudinal momentum fraction x . Within the CGC approach, many quantities become universal and simply scale with the saturation scale, the property known as a geometrical scaling. As an example of this, particle's p_t spectra in pp collisions are found to have the dependence of the type $f(p_t/Q_s)$ [10].

If the hypothesis of geometrical scaling is true, then a CGC-based estimate for the AA/pp multiplicity ratio should be energy independent (see, however, Ref. [11] discussing the DGLAP effect on the saturation scale). Yet, experiments observe a prominent growth with energy. Another observation is that the CGC-based multiplicity estimates tend to underestimate it at the LHC. In particular, Ref. [12] underestimates the observed multiplicity by approximately 35%: $dN/d\eta|_{PbPb(2.76TeV)} \simeq 1175$.

The second phase of heavy ion collision process is hydrodynamic, and it produces particles (entropy) due to finite viscosity. While the viscosity itself grows, from strongly coupled regime at the beginning of the evolution to hadronic matter at its end, and even gets very large near freezeout, the main entropy production still happens at the very beginning. This is so because the viscosity coefficient gets multiplied by flow gradients, which are fast decreasing with the evolution time. Below, we will discuss the effect of viscosity on the multiplicity growth.

III. WHEN DOES THE HYDRO STAGE START?

This question is not well posed unless we specify what exactly is meant by “hydro” and by its “start”. To define a starting moment is relatively easy: for any theory and an approximation to it, the approximation is considered as valid as long as the two deviate from each other within a preset accuracy (say one percent).

The question of defining “hydro” has different meaning and depends on what approximation is used. We will mention three cases here:

- (i) “ideal hydrodynamics” is a collective description that includes local quantities only, such as pressure and energy density. Its accuracy/validity depends on viscous corrections to this local approximation, which contain first gradients of the flow of matter.
- (ii) Navier-Stokes hydrodynamics (NS) includes these viscous terms, and its accuracy is estimated by *next* terms

involving *two* gradients.

(iii) “resummed hydrodynamics” (RH) which includes in some approximate form all higher order gradients. Accuracy of this approximation is given by deviations from first principle non-equilibrium calculations.

Obviously, as the accuracy of approximation increases from (i) to (iii), its applicability regions widens. In connection with heavy ion collision processes, it means “the beginning of the hydro stage” moves towards earlier and earlier times.

A. Conformal “resummed hydrodynamics”

When talking about all-order resummed hydro it is convenient to introduce viscosity as a momenta-dependent function. In [1] we extracted it from an AdS/CFT computed sound dispersion curve. In [13] we took a more formal approach, which lead us to propose the following model

$$\eta(\omega, k^2) = \frac{\eta_0}{1 - 1/2 k^2 - i \omega \tau_R}. \quad (\text{III.1})$$

Here $\eta_0 = 1/2$ in dimensionless units in which $2\pi T = 1$ and that corresponds to the celebrated ratio of viscosity to entropy density equal $1/4\pi$ [14]. In this units, $\tau_R = 2 - \log 2$ and is the relaxation time of the Israel-Stewart (IS) model [15]. The model (III.1) reproduces well the small ω and k expansion up to fifth order.

We consider Bjorken flow [16] as a model for the explosion. It has the simplest geometry: there is no dependence on two transverse coordinates, as well as on space-time rapidity $y = (1/2) \ln[(t-x)/(t+x)]$. What is left is a dependence on the proper time $\tau = \sqrt{t^2 - x^2}$ only. In these coordinates, the metric is $ds^2 = -d\tau^2 + \tau^2 dy^2 + d\vec{x}_\perp^2$, and we will not write any further details, as those are well known. In the Bjorken flow, there are no spatial variations ($\vec{k} = 0$) and our model (III.1) reduces back to IS. It is well known that additional non-linear terms contribute to the entropy production that is not governed by the viscosity term only. However, the entropy is produced mostly at the beginning of the expansion, when viscous terms are dominant. It is especially true for the case of very early thermalization. This is why a more or less reliable estimate of entropy production can emerge only if we know the dissipation tensor at very large ω .

Let introduce the dimensionless variable $w = \tau T$. Then, within the all order hydrodynamic approximation, the entropy production equation can be written with some “universal function” of this variable

$$\frac{dw}{d \ln \tau} = F(w), \quad (\text{III.2})$$

Solving (III.2) one finds time dependence of the temperature, from the initial time τ_i to the final (freezeout) time τ_f

$$\tau(w_f) = \tau(w_i) \exp \left[\int_{w_i}^{w_f} \frac{dw'}{F(w')} \right] \quad T(w) = w/\tau(w). \quad (\text{III.3})$$

The *final* values T_f, τ_f should be read off the experimental data (there are evidences that T_f is about the same at the RHIC and LHC while τ_f grows with E_{NN} , and hence the total entropy (multiplicity) grows too).

From these experimental data, one may use the solution and trace back to the *initial* values for the thermalization time and temperature. However, eq. (III.3) provides only one relation between the two. In the plane (τ_i, T_i) it defines a curve. (This is similar to field theory RG flows of couplings). An additional condition, to be detailed below, is needed, in order to fix the absolute values of the initial conditions.

The function $F(w)$ can be expanded in powers of $1/w$ with coefficients of the expansion being higher order viscosities. Thanks to the AdS/CFT correspondence, for conformal $\mathcal{N} = 4$ plasma the expansion terms are known up to third order [17, 18]

$$F(w)/w = \frac{2}{3} + \frac{1}{3w} \bar{\eta} - \frac{1}{3w^2} \frac{\bar{\eta}(\ln 2 - 1)}{3\pi} + \frac{15 - 2\pi^2 - 45\ln(2) + 24(\ln(2))^2}{972\pi^3 w^3} + O(1/w^4). \quad (\text{III.4})$$

The first term corresponds to the ideal hydro. The second one is NS, with $\bar{\eta} = 1/3\pi$, while the third one is the second order including non-linear terms, beyond IS. At large w the series is convergent. We will be arguing below that hydro is a reasonably good approximation for $w \geq w_0 \simeq 0.4$. For illustration purpose we give here values of these terms at w_0 , normalized to the first term:

$$(3/2)F(w_0)/w_0 = 1 + 0.1326 + 0.0107 - 0.0189. \quad (\text{III.5})$$

It is clear that the NS term is still very important. The next terms are an order of magnitude smaller. Moreover, we would like to stress the sign alternating feature of these higher order terms. As a result, being resummed, these terms contribute less than each of them separately.

To get such qualitative behavior we proposed a new and very simple “resummation model” with a new (positive) parameter α

$$F(w)/w = \frac{2}{3} + \frac{\bar{\eta}}{3(w + \alpha)}. \quad (\text{III.6})$$

This model obviously expands into a sign-alternating geometric series. The important feature is in the small w behavior, which gets regularized. One might want to relate α either to the relaxation time τ_R of IS or to the expansion terms in (III.4). However, we are to argue that the most natural choice is simply $\alpha = \bar{\eta}$: to eliminate any self heating at the early times, α shall be bigger than $\bar{\eta}$, $\alpha \geq \bar{\eta}$. $\alpha = \bar{\eta}$ looks like the optimal model choice: it leads to $T(\tau) \sim \tau^0$ at small τ , which is consistent with [19] and CGC-based estimates. This choice maximizes the amount of entropy that can be produced within the model (III.6). Larger α will drive the hydro to look more ideal. Fig.1 compares this model function with the known asymptotics at large w given by (III.4).

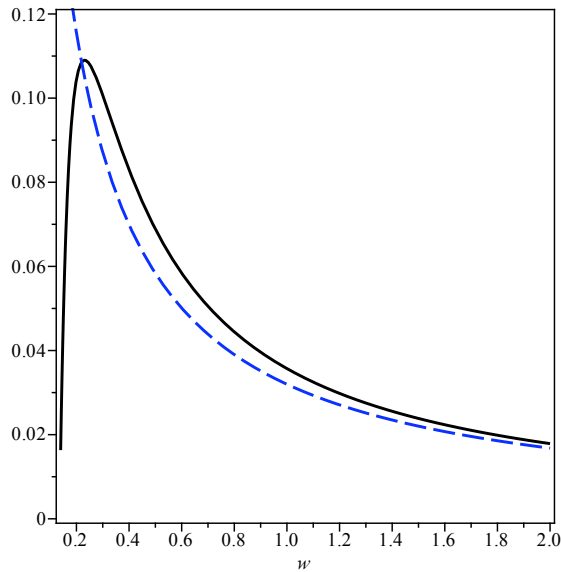


FIG. 1: (Color online) The solid line is our model for $F(w)/w - 2/3$; the dashed line is its known large w asymptotics (III.4).

B. AdS/CFT based studies of equilibration

AdS/CFT correspondence provides a possibility to study strongly coupled plasmas. We will not elaborate here on any details but will only refer to some relevant results. Following first applications of the AdS/CFT to equilibrium properties (such as equation of state) and near-equilibrium kinetic coefficients (such as viscosity), it was further realized that the duality provides a unique opportunity to study non-equilibrium problems from first principles based on well-developed gravitational tools. From the 5th dimensional perspective, a fall of an object under gravitational force is equivalent to a relaxation process, proceeding from UV to IR. This was clearly demonstrated in Ref. [20] for an elastic membrane falling under its own weight.

Without citing the full list of the AdS/CFT-based studies of non-equilibrium phenomena, we would like to refer to two recent works [18, 21], relevant for this note. Both papers address the question to what extent an sQGP explosion deduced from exact numerical solutions of the Einstein equations in AdS_5 agrees with a hydro evolution. Relying on these studies one can answer the questions posed above, namely “what is hydro?” and “when does it start?”, at least for the conformal plasma in study. As seen from Fig.3 of Ref. [21], full numerical solutions of the Einstein equations agree with the NS hydro at quite early times. Similar analysis was performed in Ref. [18]. Starting from a number

of artificial initial conditions (which, to some extent, are equivalent to introducing nuclei with arbitrary structure functions) the authors of [18] traced the exact time evolution from the gravity side. It was found that, starting from some initial w_i , the evolution of **all** trajectories converged to a universal behavior of the form (III.2). Fig.4 of this work displays this convergence and can be used to define w_i that is the “beginning of hydro”. We conclude that, depending on the accuracy requested,

$$w_i(\text{few percents}) \approx 0.40 \qquad w_i(\text{half percent}) \approx 0.65 \quad (\text{III.7})$$

One of these values provides the second relation between T_i and τ_i , which, together with (III.3), fixes the initial conditions uniquely. Obviously, our model function should be used for $w > w_i$ only. Its accuracy can be estimated from comparison with the asymptotics (III.4) Fig.1. As seen from the figure, the accuracy is about one percent or even better. It is also important to note that in both studies mentioned above, the convergence between the exact and hydro results happens when the viscosity-induced asymmetries are still very large, $O(1)$. Emergence of the ideal hydrodynamics (small asymmetries) can be also seen in those results: it happens at noticeably later times.

IV. THE ENTROPY PRODUCTION

The model (III.6) makes it possible to consider a small w limit with the function F being well regularized. Within this model, the proper time as a function of w can be found analytically:

$$\frac{\tau}{\tau_i} = \left(\frac{w}{w_i} \right)^{\frac{3\alpha}{2\alpha+\bar{\eta}}} \left(\frac{2w+2\alpha+\bar{\eta}}{2w_i+2\alpha+\bar{\eta}} \right)^{\frac{3}{2}-\frac{3\alpha}{2\alpha+\bar{\eta}}} \quad (\text{IV.1})$$

The entropy density $s = 4k_B T^3$. Assuming R , the ratio between the experimentally measured multiplicity and the pre-thermalization one, to coincide with the ratio between the finite and initial entropies, we have

$$R = \frac{s \tau}{s_i \tau_i} = \left(\frac{w}{w_i} \frac{2w_i+2\alpha+\bar{\eta}}{2w+2\alpha+\bar{\eta}} \right)^{3-\frac{6\alpha}{2\alpha+\bar{\eta}}} . \quad (\text{IV.2})$$

At the end of the evolution

$$\tau_f \sim w_f^{3/2} \rightarrow \infty .$$

R goes to its limiting value

$$R = \left(\frac{2w_i+2\alpha+\bar{\eta}}{2w_i} \right)^{3-\frac{6\alpha}{2\alpha+\bar{\eta}}} \simeq 1 + \frac{2\alpha+\bar{\eta}}{2w_i} \left(3 - \frac{6\alpha}{2\alpha+\bar{\eta}} \right) .$$

For our choice $\alpha = \bar{\eta}$

$$R = \left(\frac{2w_i+3\bar{\eta}}{2w_i} \right) \approx 1.39 \dots 1.24 , \quad (\text{IV.3})$$

where the numerical values 0.4..0.65 were used for w_i . Thus, our model can nicely recover the missing 35% in the total multiplicity production at the LHC at $E_{NN} = 2.76 \text{TeV}$. This also supports $w_i \simeq 0.5$ as the right choice for the initial condition.

More on hydro initial conditions

As we argued above, hydro evolution (III.3), supplemented by a universal value of w_i provides a means to estimate both the initial temperature T_i and initial time τ_i from the finite data. It makes sense to take as a final temperature T_f the value of 170GeV , being the QCD critical temperature. The freezeout time τ_f is not known well, neither we can be certain about our estimate of w_i . Varying these parameters we can still provide a reasonable estimate for the initial data. We do it in Fig. 2, which displays τ_i and T_i as a function of τ_f for three values of $w_i = 0, 4, 0.5, 0.6$.

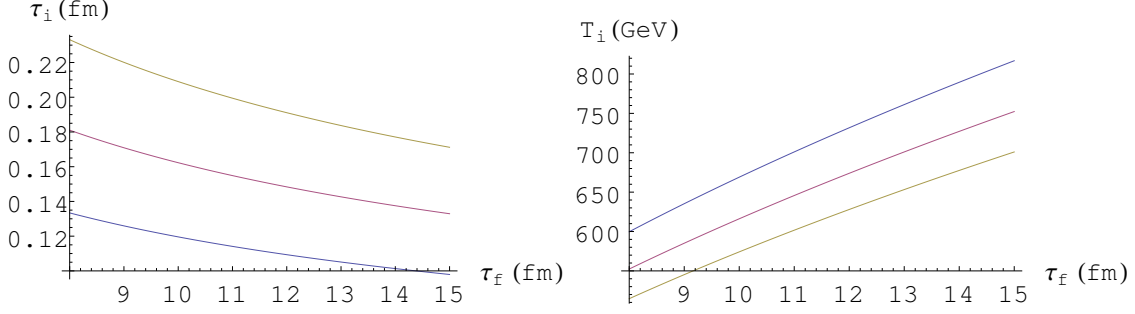


FIG. 2: The initial time and temperature as a function of the freezeout time τ_f . Three curves correspond to $w_i = 0.4$ (blue), 0.5 (red), 0.6 (yellow).

V. SUMMARY AND DISCUSSION

Inspired by the results derived via the AdS/CFT, we argued that for a rapidity-independent geometry, a non-equilibrium explosion rapidly converges to a universal hydrodynamical function $F(w)$, for which we proposed a new simple model. Using this model we estimated the amount of entropy produced in the hydrodynamic stage and found it to constitute about 30% of the total. This compliments perturbative studies such as of Ref. [12], and recovers the “missing entropy” in heavy ion collisions at the LHC. In addition, we were able to provide estimates both for the initial temperature $T_i \sim 600 - 800 \text{ GeV}$ and for the thermalization time $\tau_i \sim 0.1 - 0.2 \text{ fm}$.

The key question, of course, is to what extent our arguments and estimates are indeed applicable to QCD in real heavy ion collisions. QCD plasma is believed to be strongly coupled. From the lattice studies, thermodynamics of QCD is also known to display a near-conformal behavior ($p/T^4, \epsilon/T^4 \sim \text{const}$ for $T > 2T_c$). However, QCD is presumably not a theory with a gravity dual, and there is little information about its transport coefficients. In particular, there is no reason for the viscosity to entropy ratio to have the same universal value $1/4\pi$ as in theories with gravity duals. Indeed, presently available phenomenological estimates of the viscosity favor a larger value for the ratio: Ref. [22] obtained $\eta/s \simeq 2$ ($1/4\pi$) at the RHIC and $\eta/s \simeq 2.5$ ($1/4\pi$) at the LHC. First studies of the higher flow harmonics support these findings [3, 4]. Furthermore, this ratio is expected to increase with the temperature, because the coupling becomes weaker. Perturbative studies, such as of [23], relate η/s to an interplay between $gg \rightarrow gg$ and $gg \rightarrow ggg$ cross sections, which are $O(\alpha_s(T)^2)$ and $O(\alpha_s(T)^3)$ respectively (processes with more gluons in the final state can be also considered [24]).

Non-perturbative studies relate viscosity to an interplay between gluon-gluon and gluon-monopole scattering processes. In the latter case, there is no coupling constant in the cross section (electric charge times magnetic is integer); the monopole density, computed on the lattice, decreases with temperature. Ref. [25] predicted a rise in η/s as a function of temperature (Fig. 14 of this paper), from $\eta/s \simeq 2$ ($1/4\pi$) at $T = 2T_c$ to $\eta/s \simeq 2.6$ ($1/4\pi$) at $T = 4T_c$, roughly corresponding to the initial conditions at the RHIC and LHC.

If the QCD plasma were conformal, R would not depend on the collision energy E_{NN} , and we would obtain the same prediction for the RHIC and LHC. However, as we have noted in the beginning, experimentally it is not true. We are to speculate that this extra multiplicity observed at the LHC (relative to the RHIC normalization) may originate from the viscosity growth as a function of temperature. We further conjecture that our “universal resummed hydrodynamics” should, in some form, be valid in any theory and perhaps the same value of w_i parameter will be true in QCD. Then, the extra entropy produced, between the RHIC and LHC, can be ascribed to viscosity growth. Relying on our “resummed hydrodynamics” result we get

$$\frac{R(LHC)}{R(RHIC)} \approx 1 + \frac{3[\bar{\eta}(LHC) - \bar{\eta}(RHIC)]}{2w_i + 3\bar{\eta}(RHIC)} \quad (\text{V.1})$$

Substituting $\bar{\eta}(RHIC) \approx 2(1/3\pi)$, $w_i = 0.4$ we find that in order to get the $23 - 5.5 = 16.5\%$ (II.4) of the unaccounted extra multiplicity (double ratio) growth at LHC one would need the relative viscosity growth $[\bar{\eta}(LHC) - \bar{\eta}(RHIC)]/\bar{\eta}(RHIC) \simeq 0.4$, which is in the expected ballpark.

The ultimate knowledge about QCD transport properties will come from a systematic study of various hydrodynamical phenomena, beyond entropy production discussed in this note. The most promising ones are sound waves, already discussed in the Introduction.

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